Fixed points, stability, and intermittency in a shell model for advection of passive scalars

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We investigate the fixed points of a shell model for the turbulent advection of passive scalars introduced in Jensen, Paladin, and Vulpiani [Phys. Rev. A **45**, 7214 (1992)]. The passive scalar field is driven by the velocity field of the popular Gledzer-Ohkitani-Yamada (GOY) shell model. The scaling behavior of the static solutions is found to differ significantly from Obukhov–Corrsin scaling $\theta_n \sim k_n^{-1/3}$, which is only recovered in the limit where the diffusivity vanishes, $D \to 0$. From the eigenvalue spectrum we show that any perturbation in the scalar will always damp out, i.e., the eigenvalues of the scalar are negative and are decoupled from the eigenvalues of the velocity. We estimate Lyapunov exponents and the intermittency parameters using a definition proposed by Benzi, Paladin, Parisi, and Vulpiani [J. Phys. A **18**, 2157 (1985)]. The full model is found to be as chaotic as the GOY model, measured by the maximal Lyapunov exponent, but is more intermittent.

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I. INTRODUCTION

The origin of intermittency in fully developed turbulence is still a largely open question. Progress has been made in the somewhat simpler problem of the anomalous scaling in the passive advection of a scalar quantity (e.g., temperature or the density of a pollutant) [1,2]. Fundamental analytical results have been obtained when the advecting velocity field was assumed to be Gaussian and delta-correlated in time, the so-called Kraichnan model [3–5], and in the context of shell models [6]. Here we consider the perhaps more realistic situation where the passive scalar shell model is driven by a non-Gaussian velocity field with finite correlation time, namely the velocity field of the Gledzer-Ohkitani-Yamada (GOY) model [7,8]. In this case the model can be analyzed using standard techniques of (low) dimensional dynamical systems, e.g., the study of bifurcations, eigenvalue spectra, and Lyapunov exponents.

In the absence of convective effects the passive scalar field Θ is governed by the equation

$$\partial_t \Theta + (\mathbf{v} \cdot \boldsymbol{\nabla})\Theta = D\nabla^2\Theta + F_\Theta . \tag{1}$$

Here v is the velocity field F_Θ is an external forcing and D is the diffusion coefficient.

According to the analog of the K41 theory [9] for the passive scalar, developed by Obukhov and Corrsin [10], the structure functions

$$S_p(\mathbf{r}) = \langle |\Theta(\mathbf{x}+\mathbf{r}) - \Theta(\mathbf{x})|^p \rangle \sim l^{H_p} \tag{2}$$

(where $l = |\mathbf{r}|$) scale linearly with p, more precisely $H(p) = p/3$. Experimentally, however, one observes for $p > 3$ [11] strong deviations from this. This is usually referred to as anomalous scaling or intermittency. The deviations seem to be even more pronounced than for the structure functions of the velocity: the passive scalar is said to be more intermittent.

This paper is organized as follows: after having introduced the model, we examine in Sec. III the scaling of its fixed points and in Sec. IV their stability. These studies have already been performed for the GOY model [12–14], to which the passive scalar model is coupled. We will review these results for the sake of completeness. In Sec. V we study the full dynamics of the model and investigate its chaotic and intermittent behavior.

II. SHELL MODEL FOR PASSIVE SCALARS

Shell models appear to capture many properties of fully developed turbulent flows but are easier to study than the Navier–Stokes equations (see Ref. [15] for a review). In this paper we study the passive scalar shell model proposed in Ref. [1]. The multiscaling of the model is in good agreement with experimental data [11]. The GOY model has been studied intensively [7,8,12–22,24]; the passive scalar model has attracted somewhat less attention [6,24,25].

Both models are constructed in Fourier space, retaining only the complex modes u_n and θ_n as a representative of all modes in the shell of wave number k between $k_n = k_0\lambda^n$ and k_{n+1}. One uses the following assumptions: (i) the dissipation, respectively diffusion is represented by a linear term of the form: $-\nu k_n^2 u_n$, respectively $-Dk_n^2\theta_n$, (ii) the nonlinear terms of the form $k_n u_{n'} u_{n''}$, respectively $k_n \theta_{n'} u_{n''}$, with (iii) n' and n'' among the nearest and next-nearest neighbors of n, and (iv) in the absence of forcing and damping conservation of volume in phase space and conservation of $\Sigma_n |u_n|^2$ and $\Sigma_n |\theta_n|^2$. Moreover the scaling laws $u_n \sim k_n^{-1/3}$ and $\theta_n \sim k_n^{-1/3}$ form a fixed point of the inviscid unforced equations.

The resulting equations are [1]

$$\left(\frac{d}{dt} + \nu k_n^2\right) u_n = i(a_n k_n u_{n+1}^* u_{n+2}^* + b_n k_{n-1} u_{n-1}^* u_{n+1}^* + c_n k_{n-2} u_{n-1}^* u_{n-2}^*) + f\delta_{n,4}, \tag{3}$$

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$$\left(\frac{d}{dt}+Dk_n^2\right)\theta_n=i(e_n(u_{n-1}^*\theta_{n+1}^*-u_{n+1}^*\theta_{n-1}^*)$$
$$+g_n(u_{n-2}^*\theta_{n-1}^*+u_{n-1}^*\theta_{n-2}^*)$$
$$+h_n(u_{n+1}^*\theta_{n+2}^*+u_{n+2}^*\theta_{n+1}^*))+\overline{f}\delta_{n,4}. \tag{4}$$

A possible choice for the coefficients is

$$a_n=1 \quad b_n=-\delta \quad c_n=-(1-\delta), \tag{5}$$

$$e_n=\frac{k_n}{2} \quad g_n=-\frac{k_{n-1}}{2} \quad h_n=\frac{k_{n+1}}{2}. \tag{6}$$

The boundary conditions are

$$b_1=b_N=c_1=c_2=a_{N-1}=a_N=0, \tag{7}$$

$$e_1=e_N=g_1=g_2=h_{N-1}=h_N=0. \tag{8}$$

For the parameters, we choose for example the following standard values:

$$N=19, \quad \lambda=2, \quad k_0=\lambda^{-4}, \quad \nu=10^{-6},$$

$$f=\overline{f}=5\times10^{-3}(1+i), \quad D=10^{-6}. \tag{9}$$

The free parameter δ is related to a second quadratic invariant which for the canonical value $\delta=\frac{1}{2}$ is similar to the helicity [12].

These equations determine the evolution of the vector $(U,\Theta)=(\mathrm{Re}\,u_1,\mathrm{Im}\,u_1,\ldots,\mathrm{Re}\,u_N,\mathrm{Im}\,u_N,\mathrm{Re}\,\theta_1,\ldots,\mathrm{Im}\,\theta_N)$ and thus form a $4N$ dynamical system.

III. SCALING OF FIXED POINTS

A first step towards a full understanding of the model consists of an investigation of its static properties. In this section we examine the scaling properties of the fixed points of Eqs. (3)–(4). The major problem is to find the static solutions of Eq. (3); those of Eq. (4) can then be found easily because $\dot{\theta}$ is linear in both u and θ.

It was found in Ref. [19] that Eq. (3) in the unforced inviscid limit allows three self-similar static solutions: the trivial fixed point $u_n=0$, a ''Kolmogorov'' fixed point $u_n=k_n^{-1/3}g_1(n)$, and a ''flux-less'' fixed point $u_n=k_n^{-z}g_2(n)$, with $g_1(n)$ and $g_2(n)$ any function of period three in n and $z=(-\ln_\lambda(\delta-1)+1)/3$. The corresponding fixed points of Eq. (4) are: $\theta_n=0$, $\theta_n=k_n^{-1/3}g_1(n)$, and $\theta_n=k_n^{-(1/2)(1-z)}$. Here we will focus on the Kolmogorov fixed point which is believed to be the most important for the dynamics [19] although it was suggested that the trivial fixed point might also play a major role [21].

We note that the static solution for $u_n=u_n e^{i\phi_n}$ can be turned into real form by a change of phase. Following Schörghofer *et al.* [14] we choose the phases

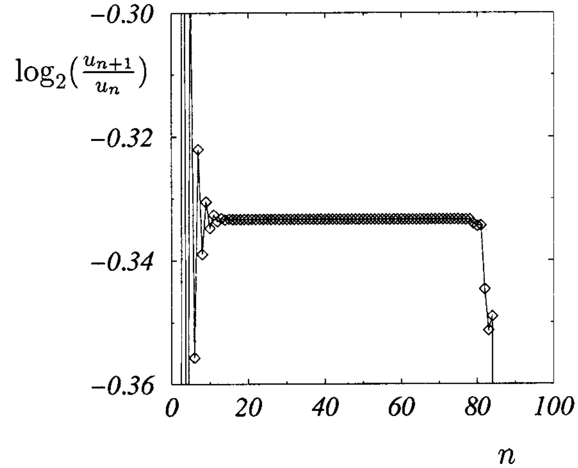

FIG. 1. Static solution of the GOY model at $\delta=0.5$ for the parameters, Eq. (11). To improve the scaling behavior, $\log_2(u_{n+1}/u_n)$ is averaged over three consecutive shells.

$$\phi_n=\begin{cases}\frac{1}{4}\pi & \text{for} \quad n=1,4,7,\ldots\\ \frac{1}{8}\pi & \text{for} \quad n=2,5,8,\ldots\\ \frac{9}{8}\pi & \text{for} \quad n=3,6,9,\ldots.\end{cases} \tag{10}$$

It can be shown that the static solution of θ picks up the same phase as that of u.

As has been observed [19], the dynamics of the system converges to the Kolmogorov fixed point for $\delta<0.379\,634$. When increasing δ the system undergoes a series of Hopf bifurcations and becomes chaotic at $\delta=0.387\,04$ [19,14,22]. In order to find the Kolmogorov fixed point one can thus vary δ in small steps and refine the solution with Newton's method [13].

To study the scaling behavior of the static solutions, we apply a much larger number of shells, using the same parameter values as Kadanoff *et al.* [14]

$$N=90, \quad \lambda=2, \quad k_0=\lambda^{-1}, \quad \nu=D=10^{-3}\times16^{-26},$$

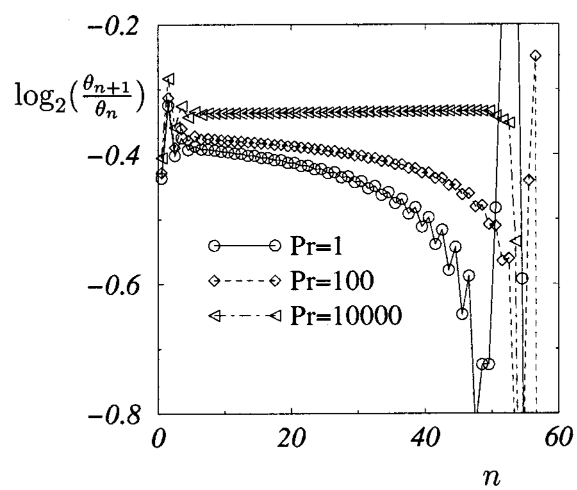

FIG. 2. Static solution, fixed point, of the passive scalar model where $\log_2(\theta_{n+1}/\theta_n)$ (averaged over three consecutive shells) is plotted vs the shell index n. Here, the number of shells is $N=61$.

$$f=\overline{f}=1. \tag{11}$$

The forcing acts in this case on the first shell.

For the static solution of u_n we obtain the same result as [14] (see Fig. 1), where $\log_2(u_{n+1}/u_n)$ is plotted versus n. It is averaged over a period three to get rid of the well known period three oscillations. The solution is seen to follow Kolmogorov scaling.

Surprisingly however, the fixed point for θ_n (with the fixed point of u_n inserted) deviates strongly from the Obukhov scaling for a finite value of the diffusivity D, as can be seen in Fig. 2. There is clearly not a well-defined power law scaling. This solution also displays the period three behavior. In the diffusive range the solution looks somewhat noisy, presumably due to the boundary conditions. It is clear that there is a ''slow'' bending in the diffusive regime but as D approaches zero, the curve becomes more and more flat and we recover the Obukhov scaling in the limit $D\to 0$. We thus note that in contrast to the velocity case where the viscosity only affects the viscous range, for the passive scalar the diffusion seems to act on the whole inertial range, at least for Prandtl numbers $\mathrm{Pr}=\nu/D\sim 1$. This might have its origin in the linear character of the problem.

IV. SCALING OF EIGENVALUE SPECTRA

Now the stability of the Kolmogorov–Obukhov fixed point is examined in terms of the eigenvalue spectra. The system [Eqs. (3)–(4)] is written as

$$\begin{cases} \dot{u}=f(u) \\ \dot{\theta}=g(u,\theta). \end{cases} \tag{12}$$

The Jacobian matrix is symbolically given by

$$J=\begin{pmatrix} \dfrac{\partial f(u)}{\partial u} & 0 \\ \dfrac{\partial g(u,\theta)}{\partial u} & \dfrac{\partial g(u,\theta)}{\partial \theta} \end{pmatrix}. \tag{13}$$

The matrix $\partial g(u,\theta)/\partial u$ will not matter for the eigenvalues of J. The eigenvalues of $\partial f(u)/\partial u$ were studied in Refs. [14] and [13]. It is most convenient to look at disturbances of phase and modulus of the velocity variable $u_n=\overline{u}_n e^{i\phi_n}$: $\overline{u}_n=u_n^{(0)}+\delta u_n$ and $\phi_n=\phi_n^{(0)}+\delta\phi_n$. One then obtains for the stability of the modulus

$$\delta\dot{u}_n=-\sum (D_{nm}+C_{nm})\delta u_m. \tag{14}$$

The matrices D_{nm} and C_{nm} are the contributions of the dissipation and cascade terms, respectively. Their expressions are

$$D_{nm}=\nu k_n^2\delta_{nm} \tag{15}$$

and

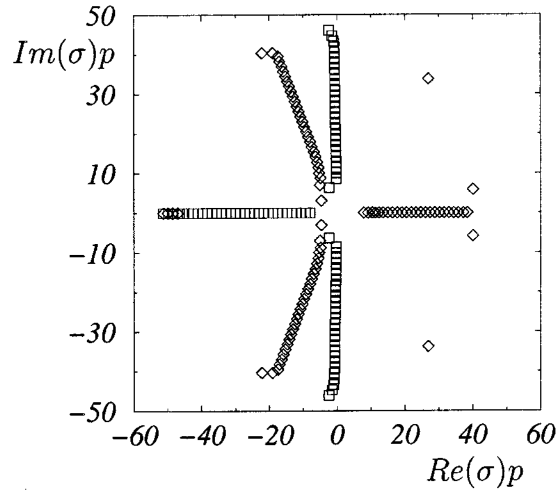

FIG. 3. Eigenvalues of phase matrix (◇) and amplitude matrix (□) for the velocity field in a polar plot for $\delta=0.5$.

$$C_{nm}=\frac{\partial}{\partial u_m}(k_n u_{n+1}u_{n+2}-\delta k_{n-1}u_{n-1}u_{n+1} -(1-\delta)k_{n-2}u_{n-1}u_{n-2}), \tag{16}$$

where the index (0) has been dropped for convenience.

The linearized equation for the variation of the phase is

$$\delta\dot{\phi}_n=-\sum (D_{nm}-C_{nm})\delta\phi_m. \tag{17}$$

Thus the stability eigenvalues of modulus and phases differ only in a minus sign in front of the cascade term. We obtain similar results as in Ref. [14]. For values of $\delta<\delta_{bif}\approx 0.37$ all the eigenvalues of both phase and modulus matrix have a negative real part. Above this value, some of the real eigenvalues of the phase matrix turn complex (in pairs) and cross the imaginary axis. At $\delta=0.5$ they have become real again, but now positive. This situation is shown in Fig. 3. The eigenvalues of the modulus matrix eventually turn positive at $\delta\approx 0.7$. In Fig. 3 we have multiplied both real and imaginary part with a factor p [14]:

$$p=\frac{\log_2(1+2^{10}|\sigma|)}{|\sigma|}. \tag{18}$$

Thus the phase remains unchanged, while the modulus is rescaled. In case the eigenvalues have constant ratios, they are evenly spaced on the plot. Thus we are able to visualize both the very small and very large eigenvalues.

It is quite trivial to generalize this method to the passive scalar in order to calculate the eigenvalues of $\partial g(u,\theta)/\partial\theta$ and one finds

$$\delta\dot{\theta}_n=-\sum_m (D_{nm}^{\theta}+B_{nm})\delta\theta_m, \tag{19}$$

where D_{nm}^{θ} is the dissipation term, very similar to the one before

$$D_{nm}^{\theta}=Dk_n^2\delta_{nm} \tag{20}$$

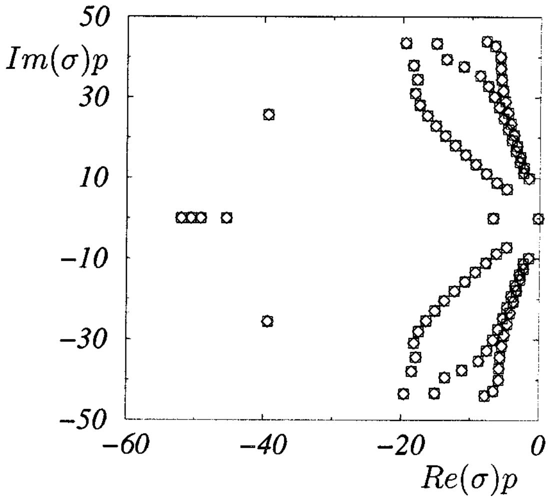

FIG. 4. Eigenvalues of phase matrix (◇) and amplitude matrix (□) for the passive field in a polar plot at $\delta = 0.5$.

and B_{nm} is the contribution of the cascade term given by

$$B_{nm} = \frac{1}{2} \frac{\partial}{\partial \theta_m} (k_n (u_{n-1}\theta_{n+1} - u_{n+1}\theta_{n-1}) - k_{n-1}(u_{n-2}\theta_{n-1} + u_{n-1}\theta_{n-2}) + k_{n+1}(u_{n+2}\theta_{n+1} + u_{n+1}\theta_{n+2})). \quad (21)$$

We see that B_{nm} does not depend on θ as the θ variation will be differentiated out.

For the phase disturbance $\delta\psi_n$ one gets

$$\delta\dot{\psi}_n = -\sum_m (D^{\theta}_{nm} - B_{nm}) \delta\psi_m . \quad (22)$$

We note that since $B_{n,n+2} = \frac{1}{2} k_{n+1} u_{n+1} = -B_{n+2,n}$ and $B_{n,n+1} = \frac{1}{2}(k_{n+1}u_{n+2} + k_n u_{n-1}) = -B_{n+1,n}$, the matrix B_{nm} is antisymmetric and its eigenvalues will be purely imaginary. The stability of phase and modulus will thus be the same. This is actually a consequence of the conservation of $\Sigma|\theta_n|^2$. In the model any cascade term $a_n u_{n+n'} \theta_{n+n''}$ has to be supplemented by a term $-a_{n-n''} u_{n+n'-n''} \theta_{n-n''}$, since the conservation implies $\Sigma \theta_n \dot{\theta}_n = 0$. The first term gives a contribution to the matrix B_{nm}: $B_{n,n+n''} = a_n u_{n+n'}$, the second $B_{n,n-n''} = -a_{n-n''} u_{n+n'-n''}$ which corresponds to $B_{n+n'',n} = -a_n u_{n+n'}$. The matrix is thus antisymmetric and this is not an artifact of the model since it stems from a conservation law also valid in a real system.

For the eigenvalues of the stability matrix $A = -(D^{\theta} + B)$ we easily obtain the inequality $\mathrm{Re}\,\sigma \leqslant 0$ since D^{θ} is diagonal with strictly positive elements. This is true whatever the driving velocity and thus one does not expect to find any bifurcations.

Indeed one finds that for all values of δ, the spectrum of eigenvalues of the phase matrix is similar to that shown in Fig. 4, where the eigenvalues are again multiplied by p. One observes evenly spaced eigenvalues organized in branches. The presence of three branches (in both upper and lower half of the complex plane) might be caused by the period three of u_n (in n). If one inserts in the matrix B_{nm} a fixed point of an imaginary model without period 3, one finds one branch in each half plane.

Let us now consider what the above imply for the dynamics of the model. At time $t=0$, starting from the particular state of the scalar $\theta_n(0)$ we impose a small perturbation: $\theta'_n(0) = \theta_n(0) + \delta\theta_n(0)$ and determines how the perturbation evolves in time, subjected to the *same* velocity field. For modulus and phase of $\delta\theta$ we have again Eqs. (19) and (22), since the matrices D^{θ} and B do not depend on θ. Because the eigenvalues of $B - D^{\theta}$ have negative real parts, the perturbation will damp out, meaning that after some time $\theta'_n(t) \sim \theta_n(t)$. This has been observed already by Crisanti *et al.* [24]. We conclude that the passive scalar cannot be chaotic in itself: chaos can only be induced by an irregular velocity field. But can it be more intermittent? We investigate this issue in some detail in Sec. V.

V. INTERMITTENCY

The term intermittency is used in different contexts and a precise mathematical definition does not exist. In turbulence one speaks of intermittency corrections to the Kolmogorov or Obukhov–Corrsin power law. In dynamical systems in general, intermittency means the presence of quiescent periods randomly interrupted by burst. It is believed that these phenomena are related: the scaling corrections should have their origin in intermittent behavior (in space and/or time) of velocity or energy dissipation. Here we ask ourselves the question whether the more pronounced deviations from classical scaling for the passive scalar are reflected by a more intermittent behavior (in time) of the passive scalar field. In order to test this we invoke a definition of intermittency for dynamical systems proposed by Benzi *et al.* [2].

First, the response function $R_t(\tau)$ is defined as the rate at which a disturbance vector $\delta\mathbf{x}(t)$ of the system $\dot{\mathbf{x}} = f(x)$ has expanded after a time τ

$$R_t(\tau) = \frac{|\delta\mathbf{x}(t+\tau)|}{|\delta\mathbf{x}(t)|}. \quad (23)$$

In chaotic systems one typically observes that $|\delta x(t+\tau)| \sim |\delta x(t)| e^{\tau\lambda}$, where λ is the maximum Lyapunov exponent. This maximum Lyaponov exponent can be calculated by averaging the *instantaneous* Lyapunov exponents $\ln R_t(\tau)/\tau$: $\lambda = \lim_{\tau\to\infty} \langle \ln R(\tau) \rangle / \tau$.

Intermittency can be thought of as connected to the fluctuations of the instantaneous Lyapunov exponent. An intermittency parameter μ is thus defined as the *variance* of $\ln R(\tau)$ [2]

$$\langle (\ln R(\tau))^2 \rangle - \langle \ln R(\tau) \rangle^2 = \mu\tau. \quad (24)$$

In shell models the maximal Lyapunov exponent turns out to be inversely proportional to the time scale of the smallest eddies and the disturbance vector (or first Lyapunov vector) is localized in the inertial range [16,23]. We thus believe the quantity μ characterizes the temporal intermittency of the cascade process.

Numerically we proceed as follows: the equations of motion (3)–(4) are numerically integrated along the equations for the disturbance vector $\dot{\delta x} = J(x)\,\delta x$, where $(x)=(u,\theta)$ and J is the Jacobian matrix. We then estimate the response function and the first two cumulants of its logarithm. From time to time the disturbance vector is renormalized in order to avoid numerical overflow [26].

Consider now the Lyapunov exponents of the full system of velocity and passive scalar field, Eqs. (3)–(4), with the parameters, Eq. (9). We distinguish how the perturbation, initially applied equally on *both* systems ($\delta u_n = \delta\theta_n$ = constant for all n) evolves independently in each of the two parts of the system. Therefore we introduce response functions that measure the expansions on the velocity part $R_t^{(u)}(\tau)$ and on the scalar part $R_t^{(\theta)}(\tau)$. They are defined as

$$R_t^{(u)}(\tau) = \frac{|\delta u(t+\tau)|}{|\delta u(t)|},\qquad R_t^{(\theta)}(\tau) = \frac{|\delta\theta(t+\tau)|}{|\delta\theta(t)|}. \tag{25}$$

The corresponding maximal Lyapunov exponents is defined as

$$\lambda^{(u)} = \frac{\langle \ln R^{(u)}(\tau)\rangle}{\tau}, \tag{26}$$

and the analog for $\lambda^{(\theta)}$. This is reminiscent to the ''Eulerian Lyapunov exponent'' and ''Lagrangian Lyapunov exponent'' introduced by Crisanti *et al.* [27,24]. However the Lagrangian behavior of the particles is only equivalent to the Eulerian passive scalar field in the case of vanishing diffusion.

The intermittency parameters are related to the variance of $\ln R(\tau)$ is the same way as before. We find numerically

$$\lambda^{(u+\theta)} \approx \lambda^{(u)} \approx \lambda^{(\theta)} = 0.165 \pm 0.002, \tag{27}$$

where we have denoted the Lyapunov exponent for the full system as: $\lambda^{(u+\theta)}$. One expects theoretically

$$\lambda^{(u+\theta)} = \max[\lambda^{(u)}, \lambda^{(\theta)}] \tag{28}$$

since the disturbance vector will evolve towards the most expanding direction. For the same reason we expect that $\lambda^{\theta} \leqslant \lambda^{u}$: $\delta\theta$ cannot grow faster than δu since that would mean that after some time we would have

$$\dot{\delta\theta} = A\,\delta\theta + \frac{\partial g(u,\theta)}{\partial u}\,\delta u \approx A\,\delta\theta. \tag{29}$$

As we have seen the matrix A has all negative eigenvalues so $\delta\theta$ would shrink: we face a contradiction and conclude that $\lambda^{\theta} \leqslant \lambda^{u}$.

Our result is obviously in agreement with both relations. For the Eulerian and Lagrangian Lyapunov exponent, one has numerically found the generic inequality $\lambda_L \geqslant \lambda_E$. The reason for which we do not find this must be the finite diffusivity because that causes the eigenvalues of A to be negative.

The intermittency parameters μ differ, on the other hand, significantly from each other. We find (for $\delta=0.5$): $\mu^{(\theta+u)} = 0.127 \pm 0.003$, $\mu^{(u)} = 0.125 \pm 0.003$, and $\mu^{(\theta)} = 0.151 \pm 0.003$. Thus the passive scalar behaves equally chaotic, but more intermittent, than the velocity. In order to understand this we write the equation for θ as follows:

$$\dot{\theta} = A(u)\,\theta + f, \tag{30}$$

where A is a matrix depending on u, whose eigenvalues σ fulfill the inequality $\mathrm{Re}\,\sigma < 0$. Thus, if u is constant in time, θ will converge to its fixed point $\theta = -A(u)^{-1} f$. Alternatively, if u fluctuates as for $\delta = 0.5$ in the GOY model, this ''fixed point'' will fluctuate as well. Moreover the convergence towards it will be irregular since the eigenvalues of $A(u)$ vary. It is therefore natural to expect that the behavior of θ is more intermittent than that of u.

VI. CONCLUSIONS AND OUTLOOK

We studied various properties of a shell model for the advection of a passive scalar. We calculate its fixed point and show that it does not follow the Obukhov–Corrsin scaling, which is only recovered in the limit $D \to 0$. This suggests that the scaling properties of the passive scalar are much more sensitive than the velocity to noninertial properties such as the dissipation and perhaps also the forcing. In experiments the scaling zones of the passive scalar are less clearly observed and some ''nonuniversality'' (with respect to large scale conditions) seems to have been observed in Ref. [28]. This however seems to be due to the lack of local isotropy, a question which cannot be addressed in the context of scalar models.

Furthermore we have calculated the eigenvalue spectrum of the fixed point and shown that the passive scalar part is stable against perturbations. This is in fact not very surprising since it follows from conservation of $\Sigma|\theta|^2$. It implies that the passive scalar field cannot be more chaotic than the velocity.

This is confirmed by measuring Lyapunov exponents of passive scalar and velocity: we find $\lambda^{(u)} \approx \lambda^{(\theta)}$. We also measure intermittency parameters of the passive scalar and velocity. The the passive scalar behaves more intermittently: $\mu^{(u)} < \mu^{(\theta)}$, which is in agreement with its more pronounced deviations from linear scaling. It remains to be seen whether these relations are ''generic'' or whether can also find $\lambda^{(\theta)} < \lambda^{(u)}$ and $\mu^{(\theta)} < \mu^{(u)}$, for instance for lower Prandtl numbers. It could also be interesting to investigate the relation between coherent structures in the GOY model [20,21] and those in the passive scalar model.

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[1] M.H. Jensen, G. Paladin, and A. Vulpiani, Phys. Rev. A **45**, 7214 (1992).
[2] R. Benzi, G. Paladin, G. Parisi, and A. Vulpiani, J. Phys. A **18**, 2157 (1985).
[3] R.H. Kraichnan, Phys. Fluids **11**, 945 (1968); Phys. Rev. Lett. **72**, 1016 (1994).
[4] K. Gawędzki and A. Kupiainen, Phys. Rev. Lett. **75**, 3834 (1995).
[5] M. Chertkov, G. Falkovich, I. Kolokolov, and V. Lebedev, Phys. Rev. E **52**, 4924 (1995).
[6] A. Wirth and L. Biferale, Phys. Rev. E **54**, 4982 (1996); R. Benzi, L. Biferale, and A. Wirth, Phys. Rev. Lett. **78**, 4926 (1997).
[7] E.B. Gledzer, Sov. Phys. Dokl. **18**, 216 (1973).
[8] M. Yamada and K. Ohkitani, J. Phys. Soc. Jpn. **56**, 4210 (1987); Prog. Theor. Phys. **79**, 1265 (1988).
[9] A.N. Kolmogorov, Dokl. Akad. Nauk SSSR **30**, 301 (1941); **32**, 16 (1941).
[10] A.M. Obukhov, Izv. Akad. Nauk SSSR, Ser. Geogr. Geofiz. **13**, 58 (1949); S. Corrsin, J. Appl. Phys. **22**, 469 (1951).
[11] G. Ruiz-Chavarria, C. Baudet, and S. Ciliberto, Physica D **99**, 369 (1996).
[12] L. Kadanoff, D. Lohse, J. Wang, and R. Benzi, Phys. Fluids **7**, 617 (1995).
[13] N. Schörghofer, L. Kadanoff, and D. Lohse, Physica D **88**, 40 (1995).
[14] L. Kadanoff, D. Lohse, and N. Schörghofer, Physica D **100**, 165 (1997).
[15] T. Bohr, M.H. Jensen, G. Paladin, and A. Vulpiani, *Dynamical Systems Approach to Turbulence* (Cambridge University Press, Cambridge, 1998).
[16] M.H. Jensen, G. Paladin, and A. Vulpiani, Phys. Rev. A **43**, 798 (1991).
[17] R. Benzi, L. Biferale, and G. Parisi, Physica D **65**, 163 (1993).
[18] D. Pisarenko, L. Biferale, D. Courvasier, U. Frisch, and M. Vergassola, Phys. Fluids A **65**, 2533 (1993).
[19] L. Biferale, A. Lambert, R. Lima, and G. Paladin, Physica D **80**, 105 (1995).
[20] J.-L. Gilson and T. Dombre, Phys. Rev. Lett. **79**, 5002 (1997).
[21] F. Okkels and M.H. Jensen, Phys. Rev. E **57**, 6643 (1998).
[22] J. Kockelkoren, F. Okkels, and M.H. Jensen, J. Stat. Phys. **93**, 833 (1998).
[23] M. Yamada and K. Ohkitani, Phys. Rev. E **57**, 6257 (1998).
[24] A. Crisanti, M.H. Jensen, G. Paladin, and A. Vulpiani, J. Phys. A **26**, 6943 (1993).
[25] K.H. Andersen and P. Muratore-Ginanneschi, Phys. Rev. E **60**, 6663 (1999).
[26] G. Benettin, L. Galgani, A. Giorgilli, and J.-M. Strelcyn, Meccanica **15**, 22 (1980).
[27] A. Crisanti, M. Falcioni, G. Paladin, and A. Vulpiani, Riv. Nuovo Cimento **14**, 1 (1991).
[28] J.-F. Pinton, F. Plaza, L. Danaila, P. Le Gal, and F. Anselmet, Physica D **122**, 187 (1998).